

One-Way Acoustic Signal Localization using Received Signal Strength

Derek Benham, Clayton Smith, Tristan Hodgins, Ashton Palacios, Philip Lundrigan, Joshua G. Mangelson

Abstract—This paper presents a low-cost and scalable localization framework for autonomous underwater vehicles using one-way acoustic communication in shallow water environments. The system relies solely on received signal strength (RSS) from four fixed acoustic beacons and a heading measurement to estimate the vehicle’s position. The signal field of each beacon is modeled independently using Gaussian Process regression, enabling a spatially-smooth data-driven representation of the signal landscape, even in environments prone to severe multipath effects. Motion is sampled probabilistically based on commanded velocity and heading measurements, then fused with RSS observations using a particle filter. Unlike conventional acoustic positioning systems that require synchronized two-way communication, our approach leverages one-way broadcasts and passive localization of the agent, reducing system complexity and enabling scalable multi-agent deployments without added infrastructure cost. Experimental results demonstrate sub-20 meter localization accuracy, outperforming a bearing only localization approach and highlighting the viability of RSS-based acoustic localization in shallow, cluttered, GPS-denied marine environments.

I. INTRODUCTION

Underwater localization remains a critical challenge, with significant implications for applications such as inspection, reconnaissance, and maritime security. Signal attenuation in water prevents submersible vehicles from utilizing common electromagnetic sensors like GPS, LiDAR, and WiFi, making robust underwater navigation an inherently complex problem. External positioning systems are a popular choice that rely on environmental hardware to aid in agent localization. Three of the most common flavors of acoustic positioning systems include short (SBL), long (LBL), and ultrashort (USBL) baseline approaches [1]. These techniques, however, rely on either two-way communication or synchronized clocks for accurate robot localization [2]. The need for two-way communication, coupled with the half-duplex nature of underwater acoustic transmissions, imposes significant overhead that severely constrains the scalability of multi-agent operations. [3].

In this paper, we report on our efforts to apply a WiFi localization technique that relies on the received signal strength (RSS) to underwater acoustic positioning systems [4]. Our approach does not require two-way communication or synchronized clocks, as is typical in common acoustic positioning systems, nor does it rely on angle of arrival

This work was funded by the Naval Sea Systems Command (NAVSEA), Naval Surface Warfare Center - Panama City Division (NSWC-PCD) under the Naval Engineering Education Consortium (NEEC) Grant Program under award number N00174-23-1-0005.

D. Benham, C. Smith, T. Hodgins, A. Palacios, J. Mangelson, are at Brigham Young University. They can be reached at {laser14, cas314, tah88, apal6981, lundrigan, mangelson}@byu.edu.

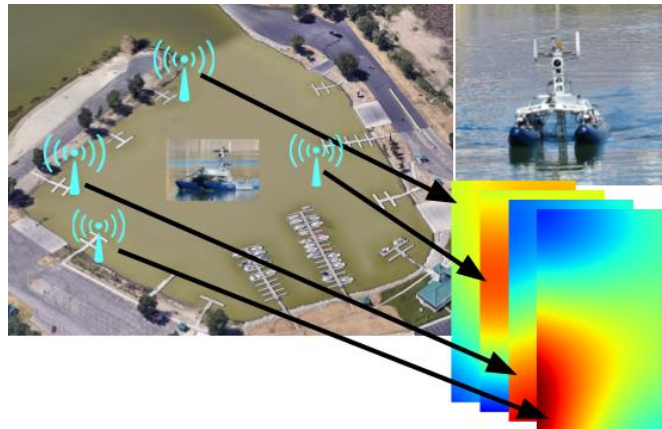


Fig. 1. Our research platform, the WAM-V 8, localizes using received signal strength values from four acoustic modems in a marina. The signal strength of each beacon is modeled individually using a Gaussian Process (GP). Motion is sampled based on the commanded velocity and a heading measurement, then fused with the received signal strength measurements through a particle filter to estimate the vehicle position.

measurements. This enables its application to any node with an acoustic hydrophone, lowering the overall cost of the system. Moreover, the system is fully passive on the agent-end, thus enabling localization of large-scale fleets. Our approach creates a prior signal strength heat map modeled with Gaussian Process (GP) regression. We then employ sequential Monte Carlo sampling to localize the agent to these heat maps. Experiments are conducted on a surface vessel equipped with a submersible sensor platform to establish GPS ground truth, with initial results demonstrating a localization accuracy of less than 20 meters, outperforming angle of arrival only methods in an environment prone to multipath propagation.

To the best of our knowledge, this is the first study to explore localization relying solely on RSS measurements from an acoustic positioning system.

The contributions of this paper include:

- Development of an RSS-based localization technique for underwater acoustic positioning systems, eliminating the need for two-way communication or synchronized clocks.
- Experimental validation of the proposed technique in a shallow-water environment, demonstrating sub-20-meter localization accuracy.
- Introduction of a cost-effective, single-node solution for underwater localization, suitable for low-cost systems and scalable to multi-agent deployments.

II. RELATED WORK

Accurate localization is fundamental for autonomous agents, particularly in environments where traditional navigation signals are unavailable or unreliable. For underwater marine vehicles, the absence of a pervasive global positioning system necessitates alternative methods to estimate their global position.

1) *Acoustic Position Systems:* Acoustic positioning systems (APS) provide underwater navigation capabilities analogous to GPS for terrestrial environments. Unlike GPS, which relies on a constellation of satellites freely accessible to any receiver with line-of-sight to the sky, APS require pre-deployed infrastructure within the operational area. This infrastructure typically consists of acoustic transducers or modems fixed at known locations in the environment, forming a local reference frame for underwater localization [5].

There are three common classes of APS: Long Baseline (LBL) [6], Short Baseline (SBL) [7], and Ultra-Short Baseline (USBL) [8] systems. These systems employ combined time-of-flight observations with time difference of arrival or phase shifts to derive the range and bearing to an acoustic source. Their primary distinctions lie in deployment scale and infrastructure complexity. Range estimation in these systems is typically performed using two-way travel time, where a signal is sent and an acknowledgment is received, allowing the system to calculate distance based on round-trip time. Alternatively, one-way travel time may be used if highly synchronized clocks are available on both ends, though this is more challenging to implement and maintain for large fleets. Additionally, each of these methods typically requires the agent being tracked to actively transmit limiting scalability and secrecy.

In a typical LBL system, several transponders are anchored to the seafloor or fixed in known positions around the perimeter of the operating area. In turn the mobile agent emits an acoustic ping to each of the surrounding transponders which then reply. The mobile agent measures the round-trip time of flight to each transponder, and through trilateration computes its own position. While LBL systems provide high positional accuracy over large areas, they require significant infrastructure and precise calibration.

SBL systems follow a similar principle but with all transducers co-located on a short array, often mounted on the hull of a support vessel or a pier. The mobile agent emits a ping, and the system computes its position using time difference of arrival and bearing information derived from the array. Because position estimation is performed externally, a return message is needed to communicate the computed location back to the mobile agent. Due to the short baseline (on the order of a few meters), the accuracy and resolution are limited compared to LBL. However, SBL systems are easier to deploy, making them suitable for short-term operations or environments where anchoring transponders is impractical.

USBL systems integrate multiple receivers into a single compact transducer head (see Fig. 2). The core innovation lies in its ability to simultaneously determine both the angle of



Fig. 2. Custom-built PVC frame used to mount static beacons in the test environment. The modular and lightweight design allows for rapid deployment and recovery of SeaTrac X150 USBL acoustic modems. Although USBL devices were used for signal collection, our RSS-based localization method does not rely on their full positioning capabilities.

arrival and the range of an acoustic signal. The tightly clustered array of receivers within the USBL transceiver exploits the subtle phase shift of the incoming sound wave across its individual elements to precisely estimate the angle of arrival (both azimuth and elevation). For distance calculation, USBL utilizes the two-way travel time of the acoustic ping, similar to the time of flight principles employed in LBL and SBL systems. This combined measurement allows for the precise, real-time localization of an underwater target; its compact size and ease of deployment make it a highly versatile and widely adopted solution for various underwater operations. When ranging information cannot be provided, a USBL can still be used to estimate its position using a bearing-only localization method [9].

While all three systems are widely used and can offer sub-meter to meter-level accuracy in ideal conditions, they share common drawbacks including (1) the need to deploy dedicated infrastructure; (2) sensitivity to environmental factors such as cluttered environments, multipath, temperature gradients, and salinity; and (3) reliance on two-way acoustic communication or tightly synchronized clocks. These constraints pose a significant challenge to scaling such systems for multi-agent deployments. In particular, if each agent were required to participate in two-way exchanges to receive its position estimate, the communication bandwidth would quickly become saturated. Moreover, achieving and maintaining precise clock synchronization across multiple mobile agents is both technically difficult and cost-prohibitive in many field environments.

2) *Signal Strength Based Localization:* There is growing interest in wireless signal-based localization [10]–[15], where

a mobile agent estimates its position using only RSS measurements. RSS measurements represent the power contained in a received radio or acoustic signal and are typically expressed in decibels relative to a reference power level. These values are readily available from most commercial wireless chipsets and sensors, making RSS an attractive modality for low-cost localization.

In air, the strength of electromagnetic (EM) signals decays logarithmically with distance and is relatively stable under ideal conditions [16]. However, in real-world environments, obstructions and multipath interference often degrade the reliability of RSS-based distance estimation [17]. Multipath interference occurs when the transmitted signal reflects off surfaces such as buildings or terrain, causing delayed copies of the signal to arrive at the receiver. The presence of overlapping or delayed signals can lead to significant fluctuations in the measured signal strength that do not correlate directly with distance, an effect that undermines the assumptions of standard path loss models. Multipath is inherently present in shallow marine environments where acoustic signals reflect not only off man-made structures such as docks and passing vessels, but also off the water surface and seafloor. Even in the absence of multipath effects, the challenge of correlating signal strength to distance in the marine domain is amplified. The strength of acoustic signals is highly sensitive to environmental factors such as water temperature, salinity, and depth, all of which can vary spatially and temporally [18], [19]. These variations make it difficult to establish a consistent mapping between signal strength and distance, complicating localization efforts in underwater and near-surface environments.

To address the multipath problem, a common technique called fingerprinting is used. Fingerprinting associates measured signals such as RSS with specific physical locations [15]. The downside to this approach is the requirement for a dense map. To overcome this, GP regression can be utilized to infer missing values [12]. Building upon these GP signal strength heat maps, [11] and [4] incorporate iterative Monte-Carlo sampling to estimate the location of a mobile agent. In this paper, we adapt these techniques to enable localization of underwater acoustic systems.

III. GAUSSIAN PROCESS BASED SIGNAL STRENGTH MAPPING

To create a prior RSS-based heatmap for localization, we employ GP regression, a non-parametric Bayesian approach well-suited for modeling spatially correlated data. GPs offer a principled way to capture both the mean and uncertainty of a continuous function given sparse and noisy measurements. These capabilities make GPs particularly effective for interpolating RSS values across an environment where physical factors may induce irregularities.

The key idea behind GP regression is that measurements taken at nearby locations are likely to be similar. This spatial correlation can be encoded using a kernel function $k(\mathbf{x}_1, \mathbf{x}_2)$, which defines the covariance between RSS values at two

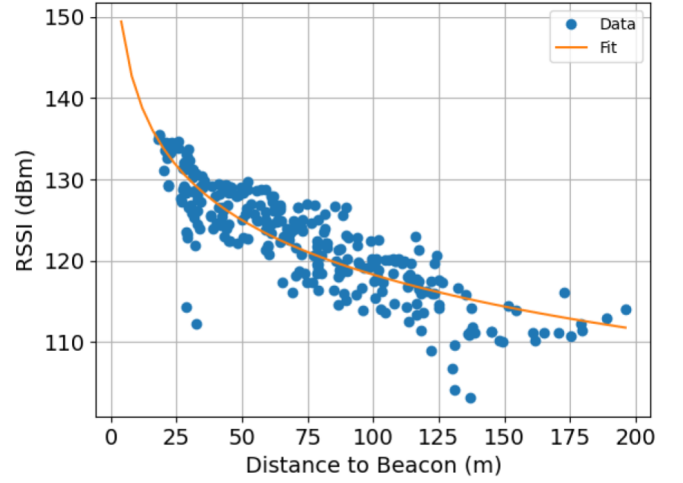


Fig. 3. Acoustic signal strength from beacon 3 is plotted against its corresponding Euclidean distance, illustrating the logarithmic decay observed in the underwater environment. This empirically derived relationship, represented by the log-normal channel model $y = a + b \ln(x)$, serves as the mean function for the Gaussian Process, providing a prior expectation of the signal field. Coefficients a and b are determined through linear least squares fitting. The data exhibits significant variability and noise, primarily due to the prevalent multipath propagation within the shallow marina test environment.

positions x_1 and x_2 . The choice of kernel determines the smoothness and generalization behavior of the model.

Following the approach established in our prior work [4], we adopt the Gaussian Radial Basis Function (RBF) kernel, defined as

$$k_{\text{RBF}}(x, x') = \sigma^2 \exp\left(-\frac{(x - x')^2}{2\ell^2}\right) \quad (1)$$

where ℓ is the lengthscale and σ is the variance. The lengthscale ℓ determines the smoothness of the resulting function by controlling how quickly it varies with input distance, while the variance σ scales the overall deviation from the mean. These kernel hyperparameters are optimized via gradient-based methods using variational inference, which updates the variational distribution to approximate the true posterior. All Gaussian Process inference is implemented using the Pyro probabilistic programming library [20].

To define the prior expectation of the signal field, Gaussian Process regression requires a mean function. While a zero-mean prior is commonly assumed in practice, we instead adopt a simplified log-normal channel model of the form $y = a + b \ln(x)$, where y represents the predicted signal strength, x is the distance to the transmitter, and a and b are coefficients learned from data in our specific test environment [21]. Although this model is relatively simple compared to those presented in [18] and [19], we find it performs sufficiently well in noisy environments (see Fig. 3) without requiring estimates of depth, salinity, or water temperature.

IV. LOCALIZATION VIA SEQUENTIAL MONTE CARLO SAMPLING

To estimate the position of the vehicle from RSS measurements, we employ a Sequential Monte Carlo Particle

Filter [22]. This filtering technique is well-suited for non-linear, non-Gaussian systems. The use of a particle filter enables robust handling of unknown initialization and allows us to simultaneously represent and track multiple hypotheses (multi-modal estimates) for the position of the vehicle. In our application, particles are used to represent the posterior distribution over 2D vehicle positions and heading. Individual GPs are trained for each acoustic beacon to model the measurement likelihoods for obtaining a specific RSS value at each location in the map.

The filter operates at 0.33 Hz, corresponding to the rate at which acoustic RSS measurements are sequentially received from individual base stations. At each time step, the particle filter performs three core operations including (1) motion propagation, (2) weight update based on the latest RSS observation, and (3) particle resampling to refine the belief over the vehicle's position.

A. Motion Propagation

To model vehicle motion, we simulate a probabilistic transition from the previous state using onboard sensor inputs. Specifically, we use the recorded heading from a dual antenna GPS and the commanded forward velocity to predict the positional change of the vehicle over a fixed time interval of one second. For each particle, a new heading is sampled from a Gaussian distribution centered at the measured heading, with added noise to reflect sensor uncertainty. The forward displacement is similarly sampled based on the commanded velocity, assuming constant speed during the interval. The resulting motion samples capture the uncertainty in vehicle motion, allowing the particle set to represent a diverse set of plausible future states.

B. Weight Update Using GP-Based Measurement Likelihood

Following the update of each particle's position, a new importance weight is assigned based on the agreement between the predicted and measured RSS from the acoustic modem. A GP trained on prior signal strength data is used to generate a predicted distribution of RSS values for each particle's location. A weighting score for each particle is then calculated by taking the inverse of the Mahalanobis distance between the observed RSS and the GP-predicted mean, with the GP-predicted variance used to normalize the residual for each measurement. This approach effectively rewards particles that lie in regions of the environment where the observed measurement is statistically probable and penalizes those where it is unlikely. Because only one base station beacon can broadcast at a time in our acoustic setup, only a single GP model (or heatmap) corresponding to that beacon is used during each measurement update.

C. Resampling

Resampling is performed with replacement, proportional to particle weights, to mitigate sample impoverishment and maintain diversity within the particle set. The resulting set of particles provides an updated belief over the vehicle position, conditioned on both the motion model and the most recent

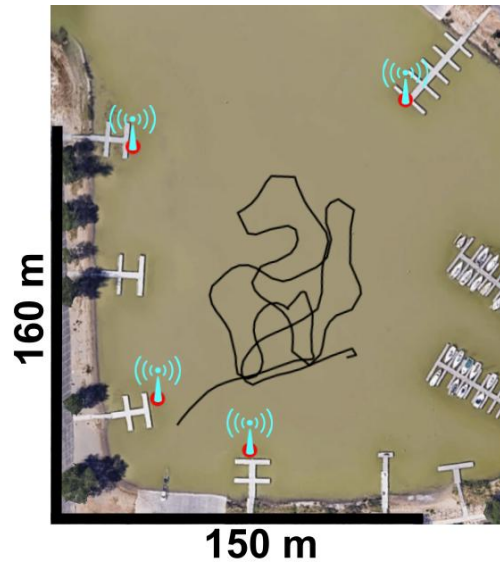


Fig. 4. Testing path (shown in black) used for evaluation. The blue antennas denote the acoustic modem locations deployed from the docks. No evaluation data was used in GP training.

RSS measurement. This process is repeated at each time step to track the vehicle as it moves through the environment.

V. RESULTS

Data for our experiments was collected at a local marina characterized by shallow waters and high levels of acoustic clutter due to surrounding docks, pilings, and other infrastructure as seen in Fig. 4. Four SeaTrac X150 USBL acoustic modems were deployed as static beacons, mounted to custom-built PVC frames and suspended approximately one meter above the seabed (see Fig. 2). The marina depth ranged between 2 and 2.5 meters resulting in a highly multipath-prone environment.

A fifth modem was mounted to the underside of our autonomous surface vehicle, the WAM-V 8, at a depth of roughly 30 centimeters below the water surface. Data was gathered using the surface vessel for convenience and to establish ground truth via GPS.

Because the static beacons were not networked together, we implemented a simple polling scheme in which the USV mounted mobile modem sequentially communicated with each of the four base stations, receiving one measurement approximately every three seconds. This asynchronous communication strategy was necessary to avoid acoustic interference. If multiple beacons transmitted simultaneously, signal collisions and packet corruption could occur. Unlike terrestrial WiFi localization, where a receiver can often observe multiple access points broadcasting on separate channels, underwater acoustic systems are bandwidth-limited. Coordinated, one-at-a-time transmissions are therefore essential for reliable operation.

Two field missions were conducted to support model training and evaluate localization performance.

To support training of the RSS GPs for each modem, the vehicle was commanded to drive in a grid pattern with 10-meter line spacing, covering the test area both vertically and

TABLE I
COMPARISON OF LOCALIZATION ACCURACY: BEARING VS. RSS-BASED
MEASUREMENT MODELS IN PARTICLE FILTER

Measurement Update	Mean (meters)	Std Dev (meters)
Bearing	21.3	2.3
RSS w/ GP	17.4	2.1

horizontally. This pattern ensured dense spatial sampling of RSS values across the marina and was used to train the GP mean function and prior map for each static beacon.

To support evaluation of the proposed localization solution, the vehicle was commanded to follow a randomized sequence of waypoints within the same area to simulate a non-structured operating pattern (see Figure 4).

A. Comparison Against a Bearing-Only Approach

To assess the performance of our RSS-based localization method, we conducted a comparative analysis using a bearing-only sensor measurement provided by the phased array of the USBL. We chose this comparison because both the RSS-based and bearing-only approaches are compatible with one-way communication and do not require synchronized clocks, making them suitable candidates for scalable, low-bandwidth systems.

The bearing-only measurement directly replaced the RSS-based sensor update step within the particle filter framework. This expected bearing measurement is computed using the known positions of the static beacons and the current heading of the vehicle, as measured onboard. All other components of the particle filter—including motion propagation, weight normalization, and resampling—were held constant to isolate the effect of the measurement modality on overall localization performance.

As presented in Table I, the RSS-based approach achieved a mean localization error of 17 meters over the full evaluation trajectory, compared to 21 meters for the bearing-only method. While both approaches are compatible with one-way communication and do not require synchronized clocks, the RSS-based update showed consistently lower error, reinforcing its utility as a practical measurement model for low-bandwidth, infrastructure-limited underwater localization.

B. Analysis of Bearing Measurement Characteristics

To characterize the impact of the shallow marina environment, particularly the prevalence of acoustic multipath propagation, we analyzed the noise associated with the bearing measurements. Fig. 5 presents two characteristics of the bearing error. Fig 5(a) displays a histogram illustrating the distribution of errors in bearing measurements to the static beacons. The error is defined as the difference between the expected azimuth to a static beacon and the actual measured azimuth. The expected azimuth measurement was calculated using the known vehicle position and heading (measured via a dual antenna GPS) and static beacon locations (measured using RTK GPS). The histogram reveals a consistent error distribution primarily within $\pm 8^\circ$, with an average absolute error of 9° and a standard deviation of 16° .

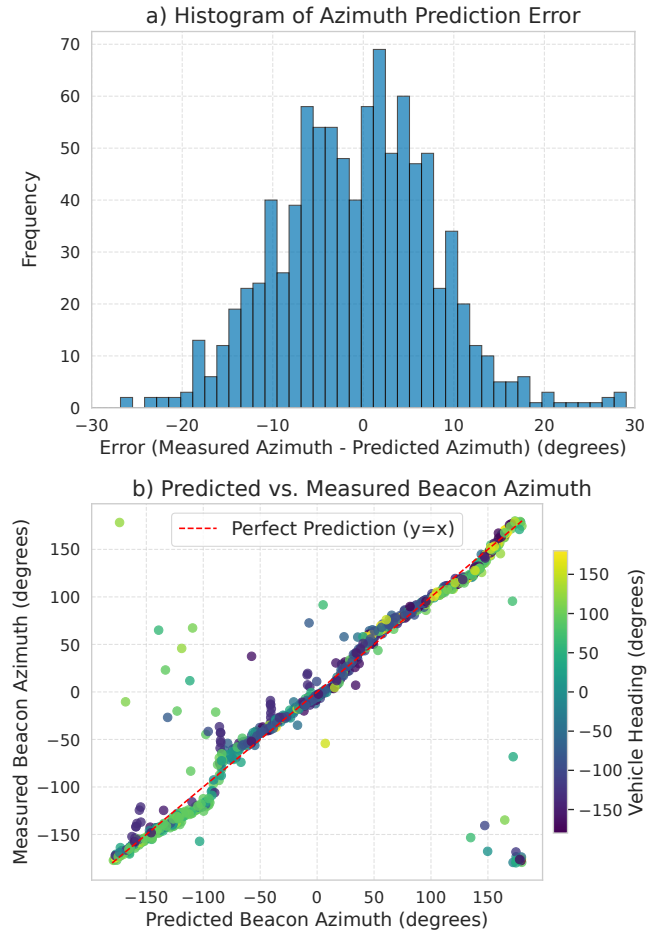


Fig. 5. Analysis of bearing measurement error. (a) Histogram illustrating the distribution of errors in bearing measurements. Error is defined as the difference between the expected beacon azimuth (derived from the measured vehicle heading and known locations of the vehicle and beacon) and the actual measured beacon azimuth. (b) Scatter plot comparing predicted beacon azimuth (x-axis) against measured beacon azimuth (y-axis), with each data point color-coded by the vehicle heading. The dashed red line represents perfect prediction ($y = x$), highlighting random measurement noise and a systematic bias that varies with azimuth, notably showing greater constant deviations near $\pm 120^\circ$ and minimal bias between $\pm 90^\circ$. Significant outliers likely due to multipath can also be seen.

When further evaluating a comparison of predicted versus measured beacon azimuths, as shown in Fig. 5(b), we see a systematic bias alongside significant measurement outliers. While random measurement noise contributes to the general scatter around the ideal $y = x$ line, the presence of sporadic, large outliers is also observed, which is consistent with multipath reflections in cluttered environments. Beyond this general noise, a clear systematic bias is evident, varying with the beacon's relative azimuth. The most pronounced bias occurs at approximately $\pm 150^\circ$ relative to the vehicle's forward direction. This specific bias coincides with the location of the vehicle's motors, which are attached to the rear of each pontoon. Although the retractable sensor plate lowers the modem beneath the pontoons, the motors and spinning propellers are still lower in the water and may introduce localized acoustic interference or flow disturbances that affect bearing measurements at these angles. It is important to note that this

motor-induced bias at $\pm 150^\circ$ does not fully account for any broader, uncharacterized systematic trends in the error, which may stem from other unknown environmental or sensor-related factors. This observation underscores a key advantage of our non-parametric GP approach: its flexibility allows it to implicitly learn and compensate for such complex, systematic biases and irregular noise patterns inherent in the acoustic environment.

VI. CONCLUSION

This work introduces what we believe to be the first method for localizing an agent using only RSS measurements from acoustic modems. The approach models the RSS of each modem using GP regression and determines absolute localization through particle filtering. In a shallow marina environment (susceptible to multipath signal propagation), the proposed method outperforms a USBL bearing-only approach, achieving an average accuracy of 17 meters RMSE.

These results support continued exploration of RSS-based acoustic localization as a scalable and low-cost alternative to traditional baseline systems, particularly in settings where two-way communication and synchronization of underwater agents is infeasible. Future work may include extending the framework to support full 3D localization by incorporating agent depth, as well as evaluating system performance in larger or more dynamic environments with multiple mobile agents.

REFERENCES

- [1] R. Jehangir, N. Purcell, and M. P. Hoover, "A smooth operator's guide to underwater sonars and acoustic devices," <https://bluerobotics.com/learn/a-smooth-operators-guide-to-underwater-sonars-and-acoustic-devices/>, Blue Robotics, June 2025.
- [2] R. M. Eustice, L. L. Whitcomb, H. Singh, and M. Grund, "Experimental results in synchronous-clock one-way-travel-time acoustic navigation for autonomous underwater vehicles," in *IEEE International Conference on Robotics and Automation*, 2007.
- [3] A. Bahr, J. J. Leonard, and M. F. Fallon, "Cooperative localization for autonomous underwater vehicles," *The International Journal of Robotics Research*, 2009.
- [4] D. Benham, A. Palacios, P. Lundrigan, and J. G. Mangelson, "Low-cost urban localization with magnetometer and LoRa technology," in *IEEE International Conference on Intelligent Robots and Systems*, October 2024.
- [5] K. Vickery, "Acoustic positioning systems. a practical overview of current systems," in *Proceedings of the Workshop on Autonomous Underwater Vehicles*, 1998.
- [6] —, "Acoustic positioning systems. new concepts-the future," in *Proceedings of the Workshop on Autonomous Underwater Vehicles*, 1998.
- [7] D. Heckman and R. Abbott, "An acoustic navigation technique," in *IEEE International Conference on Engineering in the Ocean Environment*, 1973.
- [8] M. Watson, J. Berkowitz, and M. Wapner, "Ultra-short baseline acoustic tracking system," in *IEEE International Conference on Engineering in the Ocean Environment*, 1983.
- [9] K. Fujita, T. Matsuda, and T. Maki, "Bearing only localization for multiple AUV with acoustic broadcast communication," in *International Conference on Control, Automation and Systems*, 2019.
- [10] O. Pathak, P. Palaskar, P. Rajesh, and M. Tawari, "WiFi indoor positioning system based on rssi measurements from WiFi access points-a tri-lateration approach," *International Journal of Scientific Engineering Research*, 2014.
- [11] B. Ferris, D. Hähnel, and D. Fox, "Gaussian processes for signal strength-based location estimation," in *Robotics: Science and Systems*, 2006.
- [12] S. Boonsriwai and A. Apavatjirut, "Indoor WiFi localization on mobile devices," in *International Conference on Electrical Engineering/Electronics, Computer, Telecommunications and Information Technology*, 2013.
- [13] K. Ismail, R. Liu, A. Athukorala, B. K. K. Ng, C. Yuen, and U.-X. Tan, "WiFi similarity-based odometry," *IEEE Transactions on Automation Science and Engineering*, 2023.
- [14] B. Ferris, D. Fox, and N. D. Lawrence, "WIFI-SLAM using gaussian process latent variable models," in *International Joint Conferences on Artificial Intelligence*, 2007.
- [15] S. Shang and L. Wang, "Overview of WiFi fingerprinting-based indoor positioning," *IET Communications*, 2022.
- [16] V. S. Anusha, G. Nithya, and S. N. Rao, "A comprehensive survey of electromagnetic propagation models," in *2017 International Conference on Communication and Signal Processing (ICCSP)*. IEEE, 2017.
- [17] M. Smyrniotis and S. Schön, "Multipath propagation, characterization and modeling," *Geodetic Sciences: Observations, Modeling and Applications*, 2013.
- [18] F. R. DiNapoli and R. L. Deavenport, *Numerical Models of Underwater Acoustic Propagation*. Berlin, Heidelberg: Springer Berlin Heidelberg, 1979.
- [19] W. Kuperman and P. Roux, *Underwater Acoustics*. Springer New York, 2007.
- [20] E. Bingham, J. P. Chen, M. Jankowiak, F. Obermeyer, N. Pradhan, T. Karaletsos, R. Singh, P. A. Szerlip, P. Horsfall, and N. D. Goodman, "Pyro: Deep Universal Probabilistic Programming," *J. Mach. Learn. Res.*, vol. 20, 2019.
- [21] T. S. Rappaport, *Wireless Communications: Principles and Practice*. Cambridge University Press, 2024.
- [22] S. Thrun, W. Burgard, and D. Fox, *Probabilistic Robotics*. Cambridge, MA: The MIT Press, 2005.